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Adverse Selection, Commitment and Renegotiation: Extension to and Evidence from Insurance Markets

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ABSTRACT

Under conditions of asymmetric information, commitment to long term contracts may permit markets to approach first best allocations. However, commitment is fragile and may be undermined by opportunistic behaviour, notably renegotiation. We examine the role of commitment in insurance markets. We show that fully separating strategies, when renegotiation proofed, offers the same ex ante welfare as a replication of single period equilibria. We present an alternative model (which extends Laffont and Tirole's 1990 model of procurement to address uncertainty and competition), which involves partial pooling in the first period followed by separation. This, and competing models (e.g. commitment models, single period models and "no commitment" models) have different testable predictions concerning the temporal patterns of insurer profitability. While fairly broad tests using U.S. property liability insurers favor the "no commitment" model, more focussed tests on California data suggest that some automobile insurers do use commitment to attract selective portfolios comprising disproportionate numbers of low risks.

Keywords: Adverse selection, commitment, renegotiation, insurance markets, emperical evidence of commitment.

RÉSUMÉ

En présence d'asymétrie d'information, un engagement à long terme peut permettre aux contrats d'assurance d'appromixer des solutions de premier rang. Par contre, les engagements sont fragiles et peuvent être remis en question par la renégociation. Dans cet article nous examinons le rôle des engagements dans les contrats d'assurance. Nous présentons un modèle qui étend la contribution de Laffont-Tirole sur les contrats d'acquisition afin de prendre en compte l'incertitude et la concurrence. Nous montrons qu'un contrat avec engagement partiel peut dominer une répétition de contrats à une période. De plus, nous testons empiriquement la présence d'engagements à long terme dans plusieurs marchés d'assurance aux États-Unis. Même si nous obtenons que des contrats sans engagement sont signés dans la plupart des marchés étudiés, nous vérifions que plusieurs assureurs californiens offrent des contrats avec engagement à long terme pour attirer les bons risques et pour former de meilleurs portefeuilles d'assurés.

Mots-clés: Sélection adverse, engagement à long terme, renécotiation, marchés d'assurance, évidence empirique des engagements à long terme.

INTRODUCTION

Adverse selection can be resolved in a single period by one party, the insurer, offering a menu of contracts designed to induce the other party, the policyholder, to reveal his identity by his contract selection. The classsic models are Rothschild and Stiglitz (1976) and Stiglitz (1977) who show that the use of rationing under the contract designed to appeal to low risks, can result in a separating equilibrium. While a separating equilibrium might exist in the single period setting, there is the possibility that ex ante, the welfare of all parties can be improved if they commit to a long term contract. Models by Dionne (1983) and Dionne and Lasserre (1985) show that, when the number of periods is large, experience rating can be implemented by a monopolist insurer to achieve a first best solution. Under competitive conditions, Cooper and Hayes (1987) have shown that commitment and experience rating may be used to achieve a fully separating second best equilibrium that dominates the single period strategy[1].

The basic intuition of Cooper and Hayes is appealling. Two contracts are offered. One, designed to attract high risks, provides full insurance at a premium that does not change with loss experience. But policyholders can choose an alternative contract designed to appeal to low risks, i.e. those with low probability of loss. The second contract is experience rated with low second period premiums for those who do not make a claim in the first period. Those making a first period claim face a high second period premium. This premium lottery is attractive for the low risks, given their low loss probability, but it is correspondingly unattractive for high risks who select the full insurance, flat rated contract.

Cooper and Hayes retreat from their full commitment model and consider a contract which is binding only on the insurer; policyholders are allowed to switch in the second period.

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[1]While much of Cooper and Hayes' analysis is conducted within a two period setting, they show that with a large number of periods their results converge on the first best solution in the monopoly case. This convergence likens their result to that of Dionne and Lassere (1985) (see also Gal and Landsberger 1988 and Malueg 1986, 1988). See Dionne and Doherty (1991) for an exposition of the different models.

We will refer to contracts which bind one party only as contracts with "semi commitment". Semi commitment implies an "option" feature similar to a financial option. The policyholder has the choice to renew the contract on prespecified conditions, e.g. at premiums which are conditional on prior loss experience. The insurer has a corresponding short position in this option. This feature introduces realism since insurance contracts which bind policyholders to future renewal are rarely encountered. However, the realism has a cost since it limits the ability to use experience rating as a sorting device. The possibility of policyholder switching also introduces a distinct signature to these contracts which distinguishes them from models without commitment and facilitates empirical tests of competing models. Unilateral commitment results in high first period premiums and low, below cost, expected second period premiums. The resulting intertemporal profit pattern for the insurer, declines with the age of the contract and is called "highballing".

While full commitment is desirable for the ex ante efficiency gains it promises, it is fragile and elusive; it can be undermined by opportunistic behavior. Switching represents one form of opportunistic behavior, another is renegotiation. New information can create opportunities for mutually advantageous recontracting. However, when the prospect of renegotiation is anticipated in the initial contract design, (i.e. it is made renegotiation proof), there is a loss of efficiency ex ante. Indeed, we will show that such "renegotiation proof" insurance contracts is tantamount to a replication of single period optimal contracts (i.e. without memory) and thus all efficiency gain from commitment is lost (see also Dewatripont (1989)). We will show that the efficiency gain from commitment shown by Cooper and Hayes is entirely lost if the contracts are made renegotiation proof. This does not mean that commitment is infeasible but that, to take full advantage of the efficiency gain from commitment, the parties must precommit not to renegotiate which is enforceable when the insured type is not known after the first period.

"No commitment" equilibria in insurance markets have been addressed by Kunreuther and Pauly (1985), Nilssen (1990) and others.[2] While these papers differ in important respects, they both model insurance markets in which an information asymmetry exists between policyholders and insurers. Moreover, the insurer observes the loss experience of its own policyholders and can update its priors concerning the risk type of each policyholder. These updates are not observed by rivals thus a second information asymmetry develops over time between the incumbent insurer and its rivals. Such private information is valuable. These models are characterized by "lowballing". Prices initially are low, below cost, but become profitable in future periods as the insurer extracts a "quasi rent" from the policyholder based on its use of private information[3]. Since the intertemporal profit pattern is directly opposed to that predicted in the models with commitment and switching, discriminatory tests become feasible. Nilssen's model has other features that are important for our analysis. Particularly, he shows that a pooling Nash equilibrium is possible when modelled in multiple periods. With Rothschild and Stiglitz, only separating equilibria were possible, though they might not exist.

Our purpose is to re-examine the effect of commitment in insurance markets; to examine the nature of the equilibria, whether pooling or separating, and to consider the effect of switching and renegotiation on these equilibria. Not only are we concerned with the nature of the equilibria but we are interested in extracting testable predictions from these models. Our starting point is that of Cooper and Hayes who derive fully separating equilibria (separation is achieved on initial contract selection) for both monopoly and competitive markets, both with full (bilateral) and semi (unilateral) commitment. We will show that, if contracts are rewritten

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[2]E.g. Hosios and Peters (1989). This paper differs considerably from those of Nilssen (1990) and Kunreuther/Pauly (1985). The main difference is that Hosios and Peters focus on accident under-reporting in a monopoly market which is not at issue in this and the other papers. We will assume that observable accidents are verifiable by a court so that contracts are enforceable. For example, police accident reports are available to the court and to the parties involved, but are not necessarily accessible by rival firms. This is indeed the case in some states in the U.S or provinces in Canada.

[3]See D'Arcy and Doherty (1990).

to eliminate circumstances in which renegotiation might arise, then multi period, fully separating equilibria are effectively equivalent to a replication of single period separating contracts. However, an alternative equilibrium with first period pooling and second period separation also is examined and we show that the gains from commitment may not be lost with renegotiation proofing and unilateral switching. We address both monopolistic and competitive markets. These models reveal that commitment with unilateral switching implies "highballing" which may be contrasted with the "lowballing" pricing strategies derived in the multi-period no commitment/private information models of Nilssen and Kunreuther/Pauly and the flat pricing implied by single period contracts. The final part of the paper tests for the presence of such patterns using U.S. property liability insurance data. While we cannot observe such pricing patterns directly, appropriate proxies are derived. We detect widespread evidence of "lowballing" which is indicative of no commitment. However, a more focussed test of the automobile insurance market does find evidence of "highballing" for a few insurers that underwrite predominantly low risk policyholders. These insurers appear to be using a semi commitment strategy to attract a high quality portfolio.

Our analysis is related to Laffont and Tirole's (1990) analysis of procurement contracts with renegotiation. Our competitive model shares the basic feature of Laffont and Tirole, that two contracts are offered, one selected only by high risks and the other selected by both risk types; thus the high risks can "randomize" over the two contracts and the probability of separation is non-increasing with the discount factor. However, in our application to insurance markets our model differs in several respects. We focus primarily on competitive markets and on long term contracts which bind only the insurer. Moreover, the insurance application addresses the random nature of insurable losses, and this leads to the introduction of a secondary sorting device, relating insurance premiums to past loss experience. Finally, our main difference lies in the econometric testing of this model against "no commitment" models. All the notation is collected in Appendix 1.

II. COMMITMENT AND RENEGOTIATION

Our starting point is the analysis of Cooper and Hayes (C/H). Addressing a monopoly situation, they show that the introduction of experience rating as an additional sorting device permits the insurer to increase profits compared with a single period strategy. The menu of policies offered comprises a full insurance, flat rated policy targetted at high risks (i.e. with high loss probability) and a partial insurance, experience rated policy in which the second period premium increases (decreases) if an accident (no accident) arises in the first period. Moreover, the indemnity in the experience rated policy in the second period decreases (increases) if an accident (no accident) occurs. The insurer does not know who is a low, and who is a high, risk. However, the policyholders know their risk type. Thus the premium/indemnity lottery which is introduced by experience rating is attractive to low risks given their low loss probability, but is correspondingly unattractive to high risk. C/H show that a qualitatively similar result holds under competition.

Their third case addresses a competitive situation in which the contract is binding on the insurer, but not on policyholders who have the opportunity to switch to competing insurers. The opportunity for switching, implies that the experience rating scheme must use carrots (premium reductions for favorable loss experience) rather than sticks (premiums increases following loss). Thus, to maintain the zero profit condition imposed by competition, and to prevent the high risks from selecting the experience rated contract, the first period premium on this contract is high. Low risks are willing to accept the high "up front" premium since they face a high probability of a second period premium discount. This is not true for high risks who choose the flat rated contract. The distinctive feature of this strategy is that profits will decrease with contract age for the low risks, but will be unrelated to contract age for high risks.

The full commitment strategies examined by C/H are ex ante optimal. For example, their monopoly case dominates a two fold repetition of the single period optimum of Stiglitz (1977) and their competitive case dominates a replication of Rothschild and Stiglitz (1976) assuming that a separating equilibrium does indeed exist[4]. The gains from commitment can be undermined by renegotiation (see Dewatripont (1989) and Bolton (1990)). In insurance contracting, relevant information arrives at two timepoints; at the time of initial contract selection and at the time first period losses are revealed. Once the initial contract choice is made, the insurer knows the identity of each policyholder. Thus renegotiation might arise immediately after the initial contract selection is made. We will assume a long term contract includes policies for each discrete period. As observed in many markets such as insurance, once a period commences, the contract parameters for that period are fixed. But new information may be used to renegotiate subsequent policies included in the long term contract. Thus, as with Laffont and Tirole and many others, contracts are initially written at the beginning of period one and these contracts define policies for the first and subsequent periods. The first period policies are fully enforceable as written. At the end of the first period, the insurer will have observed both the first period contract selection and the accident experience for each insured. Renegotiation of the second period policy is permitted but, if no agreement is reached, the second period policy defined in the original contract is enforced.

The full commitment contracts described by C/H do indeed offer prospect of mutually beneficial renegotiation. At the beginning of the second period, the incumbent insurer knows the identity of each policyholder and his first period loss experience. For both the monopoly case and the competitive case, at least one of the second period policies offered to the low risks entails partial insurance. Since the insurer knows the policyholder to be a low risk, then it is

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[4]Such replications of separating equilibia could only occur in markets without memory concerning first period contract choice and loss experience.

straightforward to show that there exist alternative policies offerring higher indemnities, which increase the utility of the policyholder and/or the profit of the insurer.

Figure 1 illustrates such period two policies for the monopoly case. These are offered to low risks who have a first period loss (policy G_L) and who do not have a first period loss (policy G_N). The relevant issue is that each policy is shown below the 45 degree line, indicating partial insurance. Notice that, when these policies take effect, the low and high risks already will have revealed themselves by their first period selection. With second period policies that offer less than full insurance such as G_L and G_N, there is an incentive for both parties to renegotiate. This is seen by constructing the isoprofit line for the insurer, denoted IP, which is steeper than the policyholder's utility function at any wealth level below the full insurance line:

$$-\frac{dW_A}{dW_N}\Big|_{\text{constant profit}} = \frac{1-p}{p} > \frac{(1-p)U'(W_N)}{pU'(W_A)} = -\frac{dW_A}{dW_N}\Big|_{\text{constant utility}}$$

where W_A and W_B refer to wealth in the accident and no accident states and the U's refer to marginal utilities in the respective states; p is the probability of an accident. It follows immediately, that policies such as G_N and G_L are not renegotiation proof, nor is any policy that offers less than full insurance. The prospect of renegotiation of the low risk policies affects the reservation utility of the high risks and is material to their first period contract choice. The self selection constraint for the high risks will now be more difficult to satisfy since the policies R_L and R_N are strictly preferred by high risks to G_L and G_N respectively. This follows from the shallower slope of the high risk indifference curves. This line of argument carries the convenient implication that we can restrict our search for the optimal renegotiation proof contracts with first period risk announcement to those which offer full insurance in the second period.

Proposition 1: Fully separating two period strategies that do not permit switching degenerate to replications of the single period optimal ex ante payoffs when renegotiation is permitted. This is true both for the monopoly case and the competitive case.

Proof: see Appendix 2

This result is not surprising. Anticicapted renegotiation makes the experience rated contract more attractive for the high risks and leads to a violation of the high risk self selection constraint. "Renegotiation proofing" the contract undermines the use of experience rating as a sorting device. The full advantage of commitment can be realized only if renegotiation is precluded. Figure 2 illustrates the fully separating strategy with renegotiation for low risks. In the first period, the low risks receive the wealth prospect G which is achieved through partial insurance. In the second period, the low risks receive either R_L or R_N depending on whether they suffer first period loss. While R_L and R_N are full insurance policies and independent of second period loss, they depend on first period loss by virtue of the experience rating. It is seen that the co-ordinates of these two policies are identical to the first period policy G when translated into first period loss space.

Corollary: Renegotiation proof, separating strategies with semi commitment (policyholder switching) for the competitive case cannot dominate replication of the single period payoff ex ante.

This is straightforward. Switching further limits the ability to use experience rating as a sorting device as shown by C/H. Given the additional constraint required, the strategy with switching cannot dominate the strategy without switching ex ante. Given Proposition 1, then commitment with switching cannot dominate a replication of the optimal single period payoffs. Concluding, the Cooper and Hayes contracts can only survive when there is an ex ante full commitment that precludes renegotiation. Such commitment could be forced by legal contraint or explained by the presence of net external costs to renegotiation such as reputation effects.

III. AN OPTIMAL CONTRACT WITH COMMITMENT AND RENEGOTIATION FOR AN INSURANCE MONOPOLIST

Laffont and Tirole (L/T) characterize the optimal renegotiation proof procurement contract as one which offers a choice between two contracts. The first is picked only by low cost producers. The second, a long term contract, is picked by both low and high cost producers. The second contract pools both types in the first period but offers a menu designed to induce sorting in the second period. Moreover, they demonstrate (their lemma 1) that "there is no gain from having the agent reveal more information in period one than is necessary to determine the first period allocation" (footnote 6). In this section we will briefly describe how the L/T procurement model might be adapted to the context of the insurance monopolist. While the adaptation is rather more than routine (because of the random nature of insurance losses), the qualitative features of the insurance model are quite comparable. Consequently we will not present formal analysis but simply describe the contracts.

The insurer offers a single period optimal Stigliz (1977) menu which is repeated for two periods. Policyholders can select an alternative contract which pools policyholders in the first period, but offers a separating menu in the second. High risks will randomly choose between the two strategies since the second contract is designed to offer no more rent to this group than is available in the Stiglitz contract. Low risks choose only the second contract. The insurer receives information during the first period both on the number of high risks choosing the pooling contract and on the individual loss experience of each policyholder. Both forms of information permit the insurer to revise its prior that any policyholder choosing the second contract will be a low or high risk. The second period policies under the long term contract are rated according to first period loss experience and they have the usual features of offerring full insurance to high risks and partial insurance to low risks along the high risk indifference

curve. Compared with the single period Stiglitz policies which use only rationing to sort the risk groups, this long term contract uses both experience rating and second period rationing.

IV. OPTIMAL COMPETITIVE CONTRACTS WITH SEMI COMMITMENT AND RENEGOTIATION

Under monopoly conditions the optimal contract can be organized along the lines of Laffont and Tirole as argued above. When we address competition the problem is, in some respects, simplified since competing firms limit the set of possible contracts. Firms may offer single period R/S contracts (β_H, α_H) and (β_L, α_L) in each period which achieve a separating Nash equilibrium (given appropriate proportions of low and high risks). These contracts yield zero profit to the insurer and each risk group will receive a fair price. Alternatively, the insurer may design a semi commitment contract (i.e. a renegotiation proof contract in which the second period contract is conditioned by first period loss experience but where policyholders can switch). We have already established that long term contracts which fully separate policyholders in the first period, do not dominate the single period strategy. Thus, corresponding to the monopoly case, we examine a contract which pools policyholders for the first period but requires them to reveal their identity by selection at the beginning of the second period. The two main contract choices and the policies offered under each contract are depicted in Figure 3. We first examine the nature of the second period renegotiation-proof contract. Throughout we assume that the encumbent insurer observes loss experience (there is no under-reporting of accidents) and, to prevent breakdown of rationing strategies, can write exclusive contracts (see Jaynes (1978) and Hellwig (1988) for the non enforcement case). In the second period rival firms are uniformed about first period contract choice and accident experience and can therefore offer only single period Rothschild Stiglitz contracts with prior

beliefs on λ_1. We further assume that the respective proportions of high and low risks permit a single period separating Nash equilibrium to exist.[5]

Renegotiation proof second period policies.

Assume that both parties can sign a two period contract. To improve welfare, the first period policy has to be a pooling policy defined by (α_p, β_p). At the end of the first period, the insurer offers new policies which policyholders may accept or refuse. The insurer's beliefs depend on the past experience of policyholders and on the proportion of policyholders choosing the semi commitment contract. The insurer's beliefs on any individual being a high risk, conditional on first period loss experience (k=A,N), are λ_{2A} and λ_{2N} while first period belief is denoted λ_1 $(0<\lambda_1<1)$.

The problem is to offer separating policies with rents corresponding to the transfers paid in the first period by the respective risk types. First, assume that the second period rent to low risks is zero.[6] We also note that it is not possible to penalize those sufferring a first period accident by offering a second period policy that is less attractive than the Rothschild/Stiglitz one period contract for their risk type since a competitor would simply enter the market with the single period R/S contracts. This limits, but does not preclude, the use of experience rating as a sorting device and the R/S policies define a lower bound on the second period polices offered under the semi commitment contract.

At the beginning of the second period, the insurer offers two pairs of policies to those signing the semi commitment contract: one policy pair is offered to those sufferring a first

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[5]Asheim and Nilssen (1990) and Beaudry and Poitevin (1990) re-examine the existence of separating equilibria in the presence of renegotiation.

[6] We show in Appendix 3, that the optimal contracts do not depend on this assumption. The same level of efficiency can be obtained by increasing the (negative) first period transfers to fund positive second period rents for low risks.

period accident and the other to those with no first period loss. We focus first on the no loss pair. We also know that the policies designed for high risks will offer full insurance since partial insurance would simply offer opportunity for mutually beneficial renegotiation as shown in section II above.

The problem for the insurer is to set α_{HN}, α_{LN}, and β_{LN} and the corresponding rents to solve:

$$\text{MAX } p_L U(W-D+\beta_{LN}) + (1-p_L)U(W-\alpha_{LN})$$

$$\text{s.t.} U(W-\alpha_{HN}) \geq p_H U(W-D+\beta_{LN}) + (1-p_H)U(W-\alpha_{LN})$$

$$[(1-p_L)\alpha_{LN} - p_L\beta_{LN}] \geq T^0_{LN}(x)$$

$$U(W-\alpha_{HN}) \geq U(W-\alpha^0_{HN}(x))$$

where $\alpha^0_{HN}(x)$ and $T^0_{LN}(x)$ establish the levels of rents committed in the first period policy. The rents paid in the second period depend on the proportion of high risks who select the semi commitment contract (1-x). When x=0, then $\alpha^0_{HN}(0) = \alpha_H$. Now with $T^0_{LN} = 0$ (see footnote 6), the above problem is a variation on Rothschild/Stiglitz, with respectively, a self selection constraint for high risks, a zero profit constraint for low risks and a renegotiation-proof constraint which commits a given level of second period rent to high risks.

Substituting the binding renegotiation-proof constraint into the self selection constraint and assuming that the remaining two constraints are binding, we obtain a solution that is represented in Figure 4 by the policies H_N and L_N for high and low risks respectively which is a similar to Rothschild Stiglitz but with a rent to high risks. For those high and low risks sufferring a loss, the policy pair is H_A and L_A. Recall that these are determined by the presence of competing insurers who may enter in the second period with the conventional Rothschild Stiglitz policies each offerring zero profit. The second period solution obtained here is equivalent to Laffont and Tirole's rent constrained contract. As we will see, this solution is relevant in determining the semi commitment contract.

The optimal semi commitment contract.

The semi commitment contract binds only the insurer. Whilst the insurer must offer the promised second period policies (or renegotiate to mutual advantage), the policyholder is allowed to switch to an alternative contract, which is the single period R/S contract for his risk class. This case, semi commitment, probably best describes contracts actually observed in insurance markets and therefore seems to the most promising case from which to draw testable predictions.[7] The opportunity to switch, constrains the second period rent offered to those choosing the semi commitment contract to be no less than that offered in the corresponding R/S contract. This contract must stand alone in offerring zero expected profits to the insurer. Of course, the insurer may offer both the R/S contracts and the semi commitment contract. Our problem is to design the optimal semi commitment contract.

In other respects, the competitive problem is similar to that for monopoly. A high risk may choose, either the semi commitment contract (with probability 1-x), or the high risk R/S contract (with probability x). Under the semi commitment contract, the insurer receives new information with respect to the first period loss experience and the contract selection and will revise its prior probability, λ_1, that any given individual is a high risk. A renegotiation proof

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[7]Insurance consumers are rarely contractually bound to renew policies and frequently do change their insurer when it is to their advantage. On the other side, insurers are rarely bound in the formal terms of the insurance policy to guarantee renewal. However, contracts often do have conditions which carry an implicit promise of renewal which may well be enforceable. For example, contracts which are experience rated anticipate the insurer's response to any possible claims experience by the policyholder. In this way, they implicitly pre-empt cancellation conditional on bad experience and offer the policyholder a set of future options to renew policies, each conditional on a particular loss experience. Another example relates to "claims made" policies for liability insurance. These policies cover only claims made during the policy period regardless of the timing of the event which gave rise to the claim. In contrast, "occurrence" policies cover all claims (made in the policy period or after the policy expires) with respect to events arising in the policy year. The widespread switch from occurrence to claims made policies in certain liability insurance lines in the 1980's was accompanied by an understanding that policyholders would be able to purchase subsequent policies covering latent occurrences. Such policies may be interpreted to carry an implicit long term commitment. In each example, we are not asserting that a court would enforce a long term relationship, but merely stating that an implicit relationship is understood to exist.

constraint is included in the semi commitment contract which, if it is binding, ensures that the second period policies are optimal conditional on the posterior probabilities, $\lambda_{2k}(x)$; k=A,N.

It should be clear from preceding discussion that the second period policy offered under the semi commitment policy to high risks will entail full insurance, partial insurance would simply create opportunity for mutually advantageous renegotiation. However, the second period policy for low risks may well be rationed. The first period policies under the semi commitment contract are pooling policies. Thus, at the end of the first period, the insurer still has imperfect information and may wish to use rationing to induce policyholder sorting in the second period thus the exclusivity assumption is needed.

Our main proposition is now stated.

Proposition 2: For a given probability x (x<1) that the high risks will separate, the optimal semi commitment contract under competitive conditions with policyholder switching permitted, is characterized as follows:

a. A pooling first period policy and separating second period policies. The first period pooling policy offers partial coverage.

b. Full coverage for high risks in the second period but partial coverage for low risks.

c. Experience rated second period policy for low risks. Low risks who suffer no first period loss receive more insurance in the second period while those who do suffer a loss receive the Rothschild/Stiglitz contract in the second period.

d. A second period rent is paid to high risks to participate in this contract and who suffer no first period loss. High risks who suffer loss will receive the Rothschild/Stiglitz contract in the second period. Thus high risks also are experience rated.

e. The equilibrium is characterized by positive first period expected profits for the insurer and negative second period expected profits.

The optimal semi commitment contract involves maximization of the expected utility of the low risks subject to zero profit ($\mathcal{L}_\pi$), self selection ($\mathcal{L}_{HL}$), no switching ($\mathcal{L}_{Hs}$, and $\mathcal{L}_{Ls}$) constraints and a renegotiation-proof constraint ($\mathcal{L}_{HH}$). Also, the high risk self selection constraint also embodies full insurance for high risks. To prevent switching both low and high

risks must be offered at least the expected utility in the second period from the appropriate R/S contract.[8] The self selection constraint states that high risks prefer a randomization over the high risk single period contract and the high risk semi commitment contract to the low risk semi commitment contract. However, it also ensures that high risks who select the semi commitment contract choose their designated policy in the second period when the no switching constraints are binding.[9] The decision variables are the indemnities, β's, and the premia, α's (which implicitly embody transfers and rents) for a given x. Later we discuss the optimal level of x. Complete notation is collected in Appendix 1.

(1)

$$\begin{aligned}\text{MAX}\quad & p_L U(W-D+\beta_p) + (1-p_L)U(W-\alpha_p) \\ & + \delta p_L\{\, p_L U(W-D+\beta_{LA}) + (1-p_L)U(W-\alpha_{LA})\,\} + \delta(1-p_L)\{\, p_L U(W-D+\beta_{LN}) + (1-p_L)U(W-\alpha_{LN})\,\} \\ & + \mathcal{L}_{HL}\{\,(1-x)\{\,[p_H U(W-D+\beta_p) + (1-p_H)U(W-\alpha_p)] + \delta[p_H U(W-\alpha_{HA}) + (1-p_H)U(W-\alpha_{HN})]\,\} \\ & + x[(1+\delta)U(W-\alpha_H)] - [p_H U(W-D+\beta_p) + (1-p_H)U(W-\alpha_p)] \\ & - \delta p_H\{\, p_H U(W-D+\beta_{LA}) + (1-p_H)U(W-\alpha_{LA})\,\} - \delta(1-p_H)\{\, p_H U(W-D+\beta_{LN}) + (1-p_H)U(W-\alpha_{LN})\,\}\}\end{aligned}$$

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[8]When the no switching constraints are binding, second period expected utility conditional on first period loss is set to the level that would be achieved in single period contracts.

[9]First note that the high risks who have a loss will select their appropriate second period contract ($\alpha_H=\alpha_{HA}$). This follows if the no switching constraint $\mathcal{L}_{Hs}$ is binding. To show that high risks who have no first period loss select their designated contract note that, in equilibrium, high risks randomize between the single period and long term contract implies they are indifferent between the two. Consequently, if the self selection constraint is binding then:

$$\begin{aligned}& p_H U(W-D+\beta_p) + (1-p_H)U(W-\alpha_p) + \delta[p_H U(W-\alpha_{HA}) + (1-p_H)U(W-\alpha_{HN})] \\ &= p_H U(W-D+\beta_p) + (1-p_H)U(W-\alpha_p) \\ &+ \delta p_H\{\, p_H U(W-D+\beta_{LA}) + (1-p_H)U(W-\alpha_{LA})\,\} + \delta(1-p_H)\{\, p_H U(W-D+\beta_{LN}) + (1-p_H)U(W-\alpha_{LN})\,\}\end{aligned}$$

Recalling that, with the self selection constraints binding, high risks who have a loss are indifferent between α_H and (β_{LA},α_{LA}), it follows immediately that:

$$U(W-\alpha_{HN}) = p_H U(W-D+\beta_{LN}) + (1-p_H)U(W-\alpha_{LN})$$

i.e. high risks who have no loss in the first period will self select their designated contract.

$$+ \mathcal{L}_{\pi} \{ (1-x)\lambda_1[(1-p_H)\alpha_p - p_H\beta_p] + (1-\lambda_1)[(1-p_L)\alpha_p - p_L\beta_p]$$

$$+ \delta[(1-x)\lambda_{2N}(1-p_H)(\alpha_{HN} - p_H D) + (1-\lambda_{2N})(1-p_L)((1-p_L)\alpha_{LN} - p_L\beta_{LN})]$$

$$+ \delta[(1-x)\lambda_{2A}p_H(\alpha_{HA} - p_H D) + (1-\lambda_{2A})p_L((1-p_L)\alpha_{LA} - p_L\beta_{LA})] \}$$

$$+ \mathcal{L}_{HS}\,\delta \{ U[W-\alpha_{HA}] - U[W-\alpha_H] \}$$

$$+ \mathcal{L}_{LS}\,\delta \{ p_L U(W-D+\beta_{LA}) + (1-p_L)U(W-\alpha_{LA}) - p_L U(W-D+\beta_L) - (1-p_L)U(W-\alpha_L) \}$$

$$+ \mathcal{L}_{HH}\delta \{ U(W-\alpha_{HN}) - U(W-\alpha_{HN}(x)) \}$$

The proof of Proposition 2 is contained in Appendix 3.

In Figure 4, we reconstruct the equilibrium satisfying the first order conditions shown in Appendix 3 including the various constraints. The high risk second period contracts are depicted as H_N and H_A. These are full coverage policies and reflect experience rating. The low risk pair L_N and L_A reveal partial coverage but show that the level of coverage increases if the policyholder has an accident free first year. It is seen that second period policies for low risk are breakeven but the high risk pair carries a positive expected rent. Consequently, the pooled first period policy must offer positive expected profits to the insurer in order to satisfy the zero profit constraint. We show that the pooled policy does not offer full insurance. We depict such a policy as P in the diagram. The intertemporal distribution of profits is labelled "highballing" since the initial prices and profits are loaded at the front end. Cooper and Hayes's model of competitive markets with switching and full separation yields a prediction of highballing only for the low risks; high risks choosing an alternative contract that is not experience rated and for which the profit pattern is unrelated to contract age. In selecting from the alternative R/S policy menu, high risks would receive the full insurance, fair priced policy H_A in both periods. It is immediately apparent that they would only deviate from this selection to adopt the risky strategy of pooled first period policy plus a risky second period price, if the expectation cost of this semi commitment insurance contract was lower than the actuarial fair price from the repeated R/S policy. Thus, to participate in the commitment strategy, the high risks must receive a rent. This rent will come in the form of a subsidy from the first period pooling

contract. The feasibility of the semi commitment contract depends on whether the use of joint sorting devices, (rationing and experience rating, as opposed to just rationing in the single period contracts) sufficiently reduces the cost of signalling for the low risks, that they can compensate the high risks for exposing themselves to experience rating.

As the detailed proof in Appendix 3 shows, for a given x, the optimal levels of insurance coverage do not depend on the discount factor δ. However, since the decision to separate for high risks implies intertemporal transfers, it is clear that the optimal value of x will be a function of the discount factor. Intuitively, when the discount factor is low, it is costly for insureds to pay a positive transfer in period 1 to increase insurance possibilities in period 2. Though we do not present a formal proof here (it is available on request), it may be noted that this is equivalent to L/T's theorem 3 that the high risk's probability of separating is non-increasing with the discount factor δ. Moreover, we also can show that there exists a δ^0, such that for all $\delta<\delta^0$, our optimal semi commitment contract does not dominate a sequence of Rothschild Stiglitz single period contracts. In that case, it is clear that, in marketplace without memory, the equilibrium is the Rothschild Stiglitz sequence.

The issue that now remains is whether the optimal semi commitment contract, together with the separate high risk contract, constitutes an equilibrium. Although the following discussion is not a formal proof of that our contract set constitutes a (Nash) equilibrium, we can rule out obvious alternatives as infeasible or as dominated by our contract set. First, consider the second period. From our assumptions, it is clear that the optimal second period contracts are Rothschild Stiglitz contracts with appropriate rents conditioned by accident experience or the Rothschild Stiglitz one period contract. For the first period, the possible alternatives include another pooling contract and a menu of contracts that separates risk types. We know that a pooling contract will not be chosen by low risks in the first period unless there is some commitment on the second period. This rules out any single period pooling contracts and, noting the observations on second period possibilities, we also rule out contracts with pooling in both periods. With respect to separating first period contracts, we know that a repetition of Rothschild Stiglitz contracts is dominated by our semi commitment contract, given

appropriate values of δ. We also know that long term contracts which involve first period separation do not dominate a replication of Rothschild and Stiglitz contracts. Of course, other equilibria can be derived where commitment is not allowed. We now address empirical evidence of commitment and renegotiation.

V. EVIDENCE OF LOWBALLING AND HIGHBALLING FROM PROPERTY LIABILITY INSURANCE MARKETS

In Table 1 the general features of our semi commitment model are presented alongside various other models of insurance market. The predictions of the models concerning the temporal distribution of insurer profits provide a basis for broad discrimination[10]. The tests are not perfectly discriminatory. For example, the failure to find either highballing or lowballing could indicate that markets exhibit a sequence of single period equilibria. A finding of highballing is strongly indicative of the semi commitment strategy but we are unable to discern whether this supports the fully separating model of Cooper and Hayes or the pooling-separating model developed here. A finding of lowballing is indicative of "no commitment" together with strong information asymmetries between insurers though such a result would be too crude to select between Nilssen or Kunreuther and Pauly.

Direct tests of these models require that the researcher (a) be able to separate portfolios into low and high risk groups and (b) be able to detect the pattern of prices for policyholders falling into different cohorts (where cohorts are defined by the length of time the policyholder has held a policy with a particular insurer). Both types of information raise problems. If one could isolate loss data for single risk classes (i.e. policyholders are identical in all observable respects), then differences in actual loss experience between insurers would reflect either noise or differences in the proportions of low and high risks hidden in the risk class. However, available loss data are highly aggregated over many

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[10]The predictions on the temporal profit pattern derive from the the conclusions on "lock in". All four models assume that loss experience is private information. Under those without commitment, incumbent insurers are able to capture quasi rent from this information and these models lead to consumer lock in. Under the semi commitment models, however, insurers commit to second period rents conditional on loss experience. This rules out lock in.

risk classes and, of course, the researcher has no comparative advantage over the insurer in isolating hidden information on latent policyholder characteristics. Further difficulties arise with data on prices by cohort. This data is considered by most firms to be of considerable economic value and few firms will make it available (though D'Arcy and Doherty (1990) did a limited study using a small sample of such data).

Given these data problems we have resorted to the following research design. A two stage test is undertaken. In the first stage a cross sectional regression is undertaken in which the dependent variable is the incurred losses to earned premium ratio (L/P) and independent variable is the average rate of premium growth of the insurer during the previous "n" years, G. The loss to premium ratio provides a convenient measure of pricing pattern. With highballing, the loss to premium ratio will rise with the cohort age, i.e. the longer the policy is on the insurer's books. This ratio is preferable to using underwriting profit since the latter will include expenses which are known to differ considerably between new and renewal business (see D'Arcy and Doherty for a discussion). The average rate of premium growth variable acts as a proxy for the average age of business on the insurer's books. Firms with high growth in recent years will, ceteris paribus, have a large proportion of new policies; and firms with low recent growth will have a somewhat older policy profile. To see this consider that, in the current period "t", an insurer writes a portfolio of new and renewal policies. Policies can be identified according to cohort, i.e. according to the first year they were written with that insurer. The quantity of policies written in period t belonging to cohort "t-i" is $q_{t,t-i}$, i=1,n. The insurer prices its policies by cohort to achieve a target loss to premium ratio for a policy belonging to cohort "t-i" of $(L/P)_0(1+m)^i$. The subscript zero on L/P refers to the targetted ratio for newly written policies. Clearly m>0 implies a highballing strategy and m<0 implies a lowballing strategy. Now the L/P ratio for the insurer on its whole book of business in year t, will be the weighted average of the L/P ratios for all different cohorts.

$$L/P = \sum_{i=0}^{n} (L/P)_0(1+m)^i \, w_{t,t-i}$$

where the weights are defined by:

$$w_{t,t-i} \equiv \frac{q_{t,t-n}(1+g)^i}{\sum_j q_{t,t-n}(1+g)^j}$$

The growth rate is the difference between the rate at which new policies are acquired and at which existing policies lapse. A positive growth rate will give a higher weight to newer cohorts (i.e policies recently written) and a negative growth rate will give a higher weight to early cohorts (i.e. old policies). More formally:

$$w_{t,t-i} > w_{t,t-i-1} \text{ for all i if } g>0$$

$$w_{t,t-i} < w_{t,t-i-1} \text{ for all i if } g<0$$

It follows immediately that if m>0 (i.e. highballing) then, ceteris paribus, firms with a high growth rate will have a high proportion of new business with low L/P and therefore the recorded L/P for the book of business will be low. Firms with a low growth rate will have few of the newer, more profitable, policies and the L/P for the book of business will be high. Thus, bearing in mind the inevitable noise in loss experience, the following testable form emerges:

(2) $\quad L/P = a_0 + a_1(g) + a_2(c) + \xi$

where c is a vector of control variables.

Given a mixture of high risks, for whom a flat pricing structure implies a zero slope coefficient, and low risks, for whom highballing implies a negative slope coefficient, the semi commitment models imply that the slope coefficient for the book of business as a whole will be negative. In contrast, the no commitment models of Nilssen and K/P imply a positive slope coefficient. Values insignificantly different from zero are compatible with single period pricing or simply noise in the data.

This model was estimated using data for property liability firms operating in the U.S. and appearing in the Best's tapes. The regression was estimated cross sectionally for the year 1988 using

ordinary least squares. To estimate the growth in direct premiums from the writing of new business, the following procedure was adopted. For each firm, the cumulative growth in premiums for the three years 1985-7 was extracted. The data were pruned for outliers.

Equation (2) is estimated with four control variables. The first of these is a dummy variable for the type of distribution system. Firms that sell through direct agent generally have lower expense ratios than those that use independent agents.[11] The expected sign is positive. The second control variable is the ratio of liabilites (estimated as total assets minus surplus) to premiums.[12] Lines which take longer to settle claims will exhibit a higher liability/premium ratio and thus the expected sign is positive. This leverage ratio is designed to account for differences in the lines of business written by different firms, it is used only for the results derived at the firm level. It is not used for the results from individual lines. A further control variable separates firms that are primarily reinsurers from those which are direct writers. The final control is for marketing expenses for the previous year. Such expenditures should shift the firm's demand function to the right and the predicted sign is therefore negative.

While the estimation of equation (2) reveals results that are consistent with the "no commitment" model, it does not exploit the full predictions of the competing models. Consider the semi commitment models. If similar menus of renewable, experience rated and single period contracts were offered by each firm, one would expect each firm to have a similar mixture of policies which had a flat price structure across cohorts and those which had a highballing structure. Such mixtures imply a negative coefficient for the growth variable as shown. However, if some firms were to

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[11]See D'Arcy and Doherty, 1990.

[12]This controls for difference in the line of business across firms. Some lines such as general liability or medical malpractice are so called "long tail" lines meaning that claims are usually settled a considerable time after premiums are received. Consequently, considerable investment income is generated. Conversely, lines such as auto and homeowners tend to run off quickly and little investment income is generated. Thus, equilibrium rates of underwriting profit, and thus loss ratios, would differ across lines; the higher the leverage ratio the higher the loss to premium ratio. The effect of leverage on underwriting profit is a central feature in vitually all models of competitive insurance pricing. See for example the various models presented in Cummins and Harrington (eds.) 1987.

specialize in writing different types of contracts, single period or renewable, then one would expect different values to the slope coefficient in different subsets. The slope coefficient for those specializing in renewable experience rating contracts would be lower than that for firms specializing in flat rated single period contracts. Unfortunately, this is somewhat difficult to test. We do not have available the type of qualitative data on contract types to test this directly. Another possible route would be to exploit the fact that, if some firms were specializing in renewable (single period) contracts, their policyholders would be of lower (higher) risk than the average of the rating class. Consequently the subset of insurers that have better than (worse than) average loss experience for the rating class should have lower coefficient to the growth variable.

To test the relationship requires data on losses per exposure (not losses per dollar of premiums) in similar rate classes. This data is not available in the usual published sources and what data is available is usually aggregated over many different risk classes. Thus, even if we did know the loss experience per exposure, we would not know know whether differences between insurers were due to (a) differences in the composition of rating classes between insurers or (b) arose because some insurers wrote single period contracts and others renewable experience rated contracts for the same rating class. One possible way we envisage to test this hypothesis is in using automobile liability insurance for some well defined legal jurisdiction. We do have available data on the total losses (from Best's data tapes) and on the number of vehicles insured (from the assigned risk agency AIPSO) by state. Using the California data, we have created three subgroups of insurers ranked according to the average loss per vehicle year insured. Group 1 represents those insurers whose policyholders generate the lowest loss experience per policy, group 3, the highest loss costs per policy, and group 2 is intermediate. There were 80 insurers in the sample after purging the sample of clear outliers such as firms with negative premiums or losses[13]

Before reporting these results, it might be noted that the overall results (averaged for all U.S. automobile insurers) showed evidence of lowballing. Similar evidence from a sample of the actual

[13]Such insurers are usually in the process of winding up their operations.

loss/premium ratios for seven auto insurers reported by D'Arcy and Doherty (1990) showed evidence of lowballing. In this, more focussed test, evidence of highballing is expected to be found only in subsets of insurers who exhibit low average loss per vehicle insured. Such insurers are likely to be the ones who have successfully attracted low risks by offering the renewable experience rating policies. Thus, if the highballing strategies are adopted, we should find that the slope coefficient for the premium growth variable should be negative for group 1, it should be higher for group 2 and higher still (perhaps positive) for group 3. The results documented in Table 3 confirm this pattern. Of the three subgroups, those with the lowest and highest average loss show significant slope coefficients for the growth variable. For group 3, the slope coefficient is positive, which is indicative of lowballing. For the group 1 the slope coefficient is negative; consistent with highballing. If we were to find evidence of highballing anywhere, this is where it should be. For group 2, the slope is insignificantly different from zero which confirms the monotonic pattern over the three subgroups. Finally, while our test does show evidence of semi commitment, we note that this result is consistent with both the Cooper/Hayes semi commitment model and the semi commitment model we presented above.

CONCLUSION.

Commitment to long term contracts can be used to enhance efficiency. However, the efficiency gains promised may be illusive. Long term contracts can be undermined by opportunistic behavior. We show here that extant models of commitment in insurance markets such as that of Cooper and Hayes, degenerate to replications of single period equilibria when renegotiation is allowed. However, we also derive a renegotiation proof semi commitment contract in which risk types are initially pooled, then permitted to self select into risk groups.

Whether the efficiency advantages of commitment are reaped in practice is an empirical issue. Various models, full commitment, renegotiation proof and no commitment, predict different intertemporal pricing patterns and some discriminatory testing is possible. Results based on aggregate performance of U.S. property-liability insurers suggest that pricing patterns are consistent with no

commitment models. However, more focussed tests of different groupings of automobile insurers in California, suggests that some (but not all) insurers may be using commitment to attract portfolios of predominantly low risk drivers.

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APPENDIX 1
NOTATION

i=L,H subscripts for risk groups comprising low and high risks

t=1,2 time subscripts

k=N,A subscripts to indicate whether insured has an loss or accident (A) or no accident (N) in each of the periods.The convention used is to sequence subscripts for the periods. Thus the sequence NA means no accident in period one followed by an accident in period 2.

P subscript to denote pooled first period policy

D size of insurable loss, assumed constant

(β_{ik},α_{ik}) insurance contract designed to be sold to risk group i in period 2 for given value k

β indemnity paid under insurance contract (net of premium) in loss state

α premium payable under insurance contract

δ discount factor

p_i probability of loss for risk group i

q_i number of policies sold to policyholders in group i

T_{ik} profit to the insurer for policies sold to risk group i in period 2 for given k.

U(.) utility function satisfying U'>0; U"<0 and von Neumann and Morgenstern axioms

U'(itkj) marginal utility offered to those selecting contract i, (i=L,H), in period t (t=1,2), when first period claim experience is k (k=N,A) and second period claim experience is j (j=N,A).

W starting wealth of policyholder in each period

λ_1 insurer's prior at begining of time 1 that a policyholder is a high risk

λ_{2k} is the insurer's prior at time 2 that a particular policyholder having prior loss experience k is a high risk.

$\mathcal{L}_{HL}$ Lagrangian multiplier for high risk self selection constraint

$\mathcal{L}_{HH}$ Lagrangian multiplier for renegotiation proof constraint

$\mathcal{L}_{HS}$ Lagrangian multiplier for high risk no switching constraint

$\mathcal{L}_{LS}$ Lagrangian multiplier for low risk no switching constraint

$\mathcal{L}_{\pi}$ Lagrangian multiplier for zero profit constraint

APPENDIX 2
Proof of Proposition 1

We present proof for the competitive case only. Comparable analysis can be used for the monopoly case. We first examine the optimal contracts without renegotiation.

$$
\begin{aligned}
\text{MAX} \quad & p_L U(W-D+\beta_{L1}) + (1-p_L)U(W-\alpha_{L1}) \\
& + \delta p_L \{ p_L U(W-D+\beta_{L2A}) + (1-p_L)U(W-\alpha_{L2A}) \} \\
& + \delta(1-p_L)\{p_L U(W-D+\beta_{L2N}) + (1-p_L)U(W-\alpha_{L2N})\} \\
+ \; & \mathcal{L}_{HL}\{ p_H U(W-D+\beta_{H1}) + (1-p_H)U(W-\alpha_{H1}) \\
& + \delta p_H\{ p_H U(W-D+\beta_{H2A}) + (1-p_H)U(W-\alpha_{H2A}) \} \\
& + \delta(1-p_H)\{ p_H U(W-D+\beta_{H2N}) + (1-p_H)U(W-\alpha_{H2N}) \} \\
& - p_H U(W-D+\beta_{L1}) + (1-p_H)U(W-\alpha_{L1}) \\
& - \delta p_H \{ p_H U(W-D+\beta_{L2A}) + (1-p_H)U(W-\alpha_{L2A}) \} \\
& - \delta(1-p_H)\{ p_H U(W-D+\beta_{L2N}) + (1-p_H)U(W-\alpha_{L2N}) \} \; \} \\
+ \; & \mathcal{L}_{\pi}(q_H [\{(1-p_H)\alpha_{H1} - p_H\beta_{H1}\} + \delta p_H\{(1-p_H)\alpha_{H2A} - p_H\beta_{H2A}\} + \delta(1+p_H)\{(1-p_H)\alpha_{H2N} - p_H\beta_{H2N}\}] \\
& \quad q_L [\{(1-p_L)\alpha_{L1} - p_L\beta_{L1}\} + \delta p_L\{(1-p_L)\alpha_{L2A} - p_L\beta_{L2A}\} + \delta(1+p_L)\{(1-p_L)\alpha_{L2N} - p_L\beta_{L2N}\}])
\end{aligned}
$$

As is usual, the self selection constraint is binding only for high risks while their rationality constraint is not binding. The zero profit constraint also is binding, therefore, the Lagrangian multipliers for the respective binding constraints ($\mathcal{L}_{HL}$ and $\mathcal{L}_{\pi}$) are strictly positive.

(a) Solution for High Risks

The first order conditions with respect to α_{H1} α_{H2A} α_{H2N} β_{H1}, β_{H2A}, β_{H2N} for the high risks now are identified:

(2a) $\mathcal{L}_{\pi} q_H = U'(H1N)\mathcal{L}_{HL}$
(2b) $\mathcal{L}_{\pi} q_H = U'(H2AN)\mathcal{L}_{HL}$
(2c) $\mathcal{L}_{\pi} q_H = U'(H2NN)\mathcal{L}_{HL}$
(2d) $\mathcal{L}_{\pi} q_H = U'(H1A)\mathcal{L}_{HL}$
(2e) $\mathcal{L}_{\pi} q_H = U'(H2AA)\mathcal{L}_{HL}$
(2f) $\mathcal{L}_{\pi} q_H = U'(H2NA)\mathcal{L}_{HL}$

These first order conditions clearly establish that marginal utilities, and therefore wealth, are identical for all states of nature and for each time period. Consequently, the solution involves full coverage, $\beta_H = D-\alpha_H$, and no experience rating.

(b) Solution for Low Risks

Without renegotiation proofing, the equilibrium can be determined from the first order conditions with respect to α_{L1}, α_{L2A}, α_{L2N}, β_{L1}, β_{L2A}, and β_{L2N}:

(3a) $\mathcal{L}_\pi q_L = U'(L1N)\,(1-\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL})$
(3b) $\mathcal{L}_\pi q_L = U'(L2AN)(1-\{p_H(1-p_H)/p_L(1-p_L)\}\mathcal{L}_{HL})$
(3c) $\mathcal{L}_\pi q_L = U'(L2NN)(1-\{(1-p_H)^2/(1-p_L)^2\}\mathcal{L}_{HL})$
(3d) $\mathcal{L}_\pi q_L = U'(L1A)\,(1-\{p_H/p_L\}\mathcal{L}_{HL})$
(3e) $\mathcal{L}_\pi q_L = U'(L2AA)(1-\{p_H^2/p_L^2\}\mathcal{L}_{HL})$
(3f) $\mathcal{L}_\pi q_L = U'(L2NA)(1-\{p_H(1-p_H)/p_L(1-p_L)\}\mathcal{L}_{HL})$

These lead to straightforward interpretation given $p_H>p_L$, $\mathcal{L}_\pi > 0$ and $\mathcal{L}_{HL} > 0$. $U'(L1N) < U'(L1A)$ implies partial coverage in the first period. $U'(L2AA) > U'(L2AN)$ and $U'(L2NA) > U'(L2NN)$ implies partial coverage in the second period. $U'(L2AA) > U'(L2NA)$ and $U'(L2AN) > U'(L2NN)$ implies experience rating. These results are the same as those of Cooper and Hayes.

Since risk types will have revealed themselves from first period contract choices, second period policies involving partial coverage cannot be renegotiation proof as discussed in the text. We constrain the equilibria for full coverage in the second period ($\beta_{L2A} = D-\alpha_{L2A}$; $\beta_{L2N} = D-\alpha_{L2N}$). Substituting into the maximization problem, and taking first derivatives with respect to α_{L1}, β_{L1}, α_{L2A}, and α_{L2N} gives:

(4a) $\mathcal{L}_\pi q_L = U'(L1N)\,(1-\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL})$
(4b) $\mathcal{L}_\pi q_L = U'(L1A)\,(1-\{p_H/p_L\}\mathcal{L}_{HL})$
(4a) $\mathcal{L}_\pi q_L = U'(L2N)\,(1-\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL})$
(4b) $\mathcal{L}_\pi q_L = U'(L2A)\,(1-\{p_H/p_L\}\mathcal{L}_{HL})$

Thus, given $p_H>p_L$, $\mathcal{L}_\pi > 0$ and $\mathcal{L}_{HL} > 0$, then $U'(L1N) = U'(L2N)$, $U'(L1A) = U'(L2A)$ and $U'(L1N) < U'(L1A)$. The first period contract for the low risks involves partial insurance; wealth is conditional on first period loss. The first period wealth is exactly replicated in the second period. However, while independent of second period loss since there is full coverage, second period wealth is conditional on first period loss by virtue of the experience rating. In others words, second period wealth is not only identical to, but perfectly correlated with, first period wealth.

Since each party has identical wealth in both periods, the two period profit maximising strategy must exactly replicate the single period profit maximising strategy offered in the first period (Stiglitz 1977). A qualification is needed. The second period contract offered to low risks is not identical to the first period contract. The second period contract is a full coverage, experience rated contract. However, the second period contract offers identical wealth to the first period contract, conditional on first period loss. It therefore must offer the same expected utility and expected profit as a two fold repetition of the optimal single period contracts[1].

[1]Note that, if utility were not assumed to be time separable, then a replication of two single period contracts would not necessarily offer the same expected utility as the two period experience rated contract. Since the latter yields perfectly correlated wealth (conditional on first period loss), then indifference between the two contracts implies indifference to intertemporal correlation. This is a strong assumption. Indeed financial markets have developed many instruments for intertemporal hedging.

APPENDIX 3
PROOF OF PROPOSITION 2.

(a) All constraints are binding.

We first consider whether the various constraints introduced into this problem are binding. Disposition of the self selection and zero profit constraints is routine for this type of problem. Competition and freedom of entry without transaction cost ensures that profit for each type of contract will be driven to zero. The binding of the high risk self selection constraint is necessary to ensure that high risks do not select the low risk contract. As this constraint is written, we also observe that high risks are indifferent between their contract and a two period repetition of the Rothschild and Stiglitz contract. Thus we accept $\mathcal{L}_{HL}$ and $\mathcal{L}_{\pi}$ as binding constraints. We now turn to the renegotiation proof constraint $\mathcal{L}_{HH}$ and the no switching constraints $\mathcal{L}_{LS}$ and $\mathcal{L}_{HS}$.

LEMMA 1: The renegotiation-proof constraint is binding; $U(W-\alpha_{HN}) = U(W-\alpha_{HN}(x))$

Proof : Under competition (zero profit) the insurer cannot offer a rent to the high risk individuals greater than that corresponding to the transfers paid in the first period which are determined by the value of x. Moreover, the insurer cannot offer a higher premium than $\alpha_{HN}(x)$ to the high risk individuals when the self selection constraint is binding (see footnote 9 for more details).

LEMMA 2: The no switching constraints are binding.

Proof : Given competition, $U(W-\alpha_{HA})$ cannot be lower than $U(W-\alpha_H)$ and similarly, $p_L U(W-D-\beta_{LA}) + (1-p_L)U(W-\alpha_{LA})$ cannot be lower than $p_L U(W-D-\beta_L) + (1-p_L)U(W-\alpha_L)$ which is the level of welfare corresponding to the Rothschild-Stiglitz single period contract. We can also show that there is no efficiency gain from relaxing these constraints by permitting strictly positive rents in the second period. In other words, the welfare of all parties ex ante, is unaffected by fixing the second period rents to those having a first period accident to zero. Since the formal proof needs further notation this will be presented at the end of this appendix in (f) below after the main results.

Finally, before proceeding with the main results, a similar line of proof can be used to show that second period policies for low risks carry zero profit.

In Figure 4, we illustrate the low risk second period policy pair, L_N and L_A on the low risk fair price line. Accordingly, these policies are priced at their actuarially fair price and offer the insurer zero expected profit. We address whether there is an eficiency gain from increasing the first period transfer from the pooled policy in order to offer second period rent to low risks, i.e. moving the points L_A and L_N to the right of the fair price line. A demonstration that there is no efficiency gain is straightforward but tedious. This takes similar form to the demonstration that there is no efficiency gain from increasing second period rents to those having first period accidents discussed above. We simply sketch out the direction and leave it to the reader to verify.

An additional rent to low risks must be paid from the first period pooled policy such that the zero profit constraint for the semi commitment contract is not violated. However, because some high risks also purchase the pooled policy, they too will pay the higher first period transfer.

To ensure that the self selection constraint is not violated, high risk must therefore receive higher second period rent to compensate. The introduction of the second period rent to low risks, together with the corresponding transfers and rents in the pooled policy, can be shown in equation 1 in the text. Taking the appropriate derivatives with respect to the low risk rent, and substituting into the first order conditions 5.1 -5.4 below, reveals that this second period rent to low risks offers no welfare gain.

Since the $\mathcal{L}_{HH}$, $\mathcal{L}_{LS}$ $\mathcal{L}_{HS}$ constraints are binding, it follows that $\alpha_{HN}=\alpha_{HN}(x)$, $\alpha_{HA}=\alpha_{H}$, $\beta_{LA}=\beta_{L}$, and $\alpha_{LA}=\alpha_{L}$. To simplify the derivation of the following results we substitute these values into the problem. The maximization problem with respect to α_{P}, β_{P}, α_{LN}, and β_{LN} can be separated into two separate maximization problems. We first consider the optimal choice of α_{LN}, and β_{LN} for those who had no first period accident (i.e. the state of nature for the first period has been revealed and the optimization problem is now reduced to the maximization of a single period contract given the rent committed to in the first period). The first order conditions are:

5.1 (α_{LN}): $U'(L2NN)\,[1-\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL}]-\mathcal{L}_{\pi}=0$

5.2 (β_{LN}): $U'(L2NA)\,[1-\{p_H/p_L\}\mathcal{L}_{HL}]-\mathcal{L}_{\pi}=0$

(b) Partial second period coverage for low risks.

From 5.1 and 5.2:

$$\frac{U'(L2NN)}{U'(L2NA)} = \frac{1-\{p_H/p_L\}\mathcal{L}_{HL}}{1-\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL}}$$

Since $p_H>p_L$ then $U'(L2NN) < U'(L2NA)$ which implies the following ordering of wealth in those states, $W(L2NN) > W(L2NA)$. This solution corresponds to the rent constrained solution in the renegotiation proof second period policy in the first subsection of IV. The level of insurance to low risks in the second period depends on the rent committed to high risks in the first period. Note that the second period policy for high risks who have no first period accident is determined by the self selection constraint as argued in first subsection of IV. The high and low risk policies are shown as H_N and L_N in Figure 4.

(c) Pooling first period policy offers partial coverage.

Maximization with respect to α_P and β_P yields:

5.3 (β_P): $U'(P1A)\,[1+(p_H/p_L)\mathcal{L}_{HL}]-\mathcal{L}_{\pi}[(1-\lambda_1)+(1-x)\lambda_1 p_H/p_L]=0$

5.4 (α_P): $U'(P1N)\,[1+\{(1-p_H)/(1-p_L)\}\mathcal{L}_{HL}]-\mathcal{L}_{\pi}[(1-\lambda_1)+(1-x)\lambda_1\{(1-p_H)/(1-p_L)\}]=0$

From 5.3 and 5.4 we obtain:

$$U'(P1A)-U'(P1N)$$

$$=\mathcal{L}_{HL}x[U'(P1A)(p_H/p_L)-U'(P1N)\{(1-p_H)/(1-p_L)\}]+\lambda_1\mathcal{L}_{\pi}(1-x)[(p_H/p_L)-(1-p_H)/(1-p_L)]$$

When x=0, we obtain the pure pooling solution corresponding to the maximization of the low risk individual's utility; we denote this solution β_{P0}

When x=1, we verify the Rothschild Stiglitz solution that $\alpha_p=\alpha_L$ and $\beta_p=\beta_L$, which we describe as pure separation.

Otherwise, $0<x<1$, we have a mixture of the two cases, (i.e. some high risks buy the pooled contract but others do not) which implies that β_P is a decreasing function of x (the probability that the bads risks separate in the first period). Consequently, $\alpha_P(x)$ also is a decreasing function of x. When x=1, $\beta_P=\beta_L$. In this case, no rent is paid to high risk individuals in the second period since the long run contract is a repetition of the Rothschild/Stiglitz contract. Intuitively, the high risk individuals have full insurance coverage and do not need any compensation.

When x=0, (pure pooling), the price line for the pooling policy will have the same slope as the single period no profit pooling contract. When x=1, (pure separation), the pooling policy will have a price line equal to the slope of the single period low risk policy. When $0<x<1$, the price line will have an intermediate slope. However, for all $x<1$, the first period policy must inlcude a transfer, (shown in Figure 4 by shifting the pooling price line towards the origin) necessary to finance the second period rents to high risks. The net rent to high risks must be positive to compensate them for the risk inherrent in experience rating of the semi commitment contract. However, this system of rents and transfers permits an efficiency gain since low risks may have more converage than under a repetition of the single period Rothschild and Stiglitz separating strategy.

(d). Experience rating and positive second period rents for high risks.

From the preceeding discussion, when $x<1$, the high risk individual receives a rent in the second period in order to induce first period pooling and self selection. Since we have already argued that the no switching constraints are binding, and that low risk insurance is fairly priced, we can verify from the zero profit constraint that in equilibrium:

$$(1-x)\lambda_1[(1-p_H)\alpha_P(x) - p_H\beta_P(x)] + (1-\lambda_1)[(1-p_L)\alpha_P(x) - p_L\beta_P(x)] + \delta[(1-x)\lambda_{2N}(x)(1-p_H)(\alpha_{HN}(x) - p_H D]$$

$$= 0$$

or, equivalently,

$$\alpha_{HN}(x) = p_H D - \frac{T(x)}{\delta(1-x)\lambda_{2N}(x)(1-p_H)}$$

where:

$$T(x) \equiv (1-x)\lambda_1[(1-p_H)\alpha_P(x) - p_H\beta_P(x)] + (1-\lambda_1)[(1-p_L)\alpha_P(x) - p_L\beta_P(x)]$$

is the net transfer paid to the insurer in the first period. Clearly, T(x)=0 when x=1 (pure separation) and $T'(x)<0$. This last result comes from the self selection constraint. If we wish to keep the high risk individuals indifferent between the two contracts, T(x) must increase when some high risks choose to pool ($x<1$). Differentiation of the above expression with respect to x yields that $\alpha'_{HN}(x) > 0$ which implies that, for any $x<1$, $\alpha_{HN}(x) < \alpha_{HA} = \alpha_H$. Consequently, when $x<1$, the high risk individuals are experience rated. This implies that low risks also are experience rated since, at equlibrium:

$$U(W-\alpha_{HN}(x)) = p_H U(W-D+\beta_{LN}) + (1-p_H)U(W-\alpha_{LN})$$

(f). Further discussion on the result that the no switching constraints are binding

When these constraints bind, the pair of second period, full insurance, experience rated policies for high risks who choose the semi commitment contract is H_A and H_N as shown in Figure 4. Morover, the pair of policies for those selecting the semi commitment contract but having a first period loss is H_A and L_A for high and low risks respectively, which is simply the Rothschild and Stiglitz one period menu.

Consider now that the no switching constraints do not bind by introducing a second period rent of K both to those high risks having a loss and those that do not. Thus the high risk policy pair is (each policy defined by (indemnity, premium)):

$$(D, \alpha_H - K) \quad \text{and} \quad (D, \alpha_{HN} - K)$$

The additional rent K cannot come from the second period low risk policies since these will be fairly priced. However, the increased second period rent to high risks permits more coverage to be offered to the low risks while satisfying the requirements for second period self selection. Thus the corresponding low risk policies are defined such that high risks are indifferent between the low and high risk policies:

$$(D, \alpha_H - K) \sim (\beta'_{LA}, \alpha_{LA} - g_1(K))$$

$$(D, \alpha_{HN} - K) \sim (\beta'_{LN}, \alpha_{LN} - g_2(K))$$

where, because of fair pricing, g_1 and g_2 must satisfy:

$$\beta'_{LA} \equiv [(1-p_L)/p_L][\alpha_{LA} - g_1(K)] \text{ and } \beta'_{LN} \equiv [(1-p_L)/p_L][\alpha_{LN} - g_2(K)]$$

The additional rent from the second period high risk policies must come from the first period pooled policy. This policy now includes a transfer f(K) as follows:

$$(\beta_P - f(K), \alpha_P + f(K))$$

This whole set of policies defined by the new rent K, can now be substituted into equation 1 and the first derivatives taken with respect to K. This is straightforward but messy so we will not reproduce the analysis here. Substitution of the first order conditions 5.1 through 5.4 into this derivative reveals that all terms cancel out and the derivative with respect to K is zero. This shows that there is no welfare gain from relaxing the no switching constraints by introducing the additional rent.

TABLE 1

Characteristics	Kunreuther and Pauly single period equilibria No commitment	Cooper and Hayes Semi commitment	Nilssen No commitment	Dionne/Doherty Semi-commitment & renegotiation
Price-quantity contracts	no	yes	yes	yes
Commitment to more than one period				
insurer	no	yes	no	yes
insured	no	no	no	no
Insurers observe				
total contract choice	no	yes	yes	yes
accidents	no (claims only)	yes	yes	yes
Rivals observe				
contract choices	no	yes in current period no for entrants	yes	yes in current period no for entrants
loss experience	no	no	no	no
Type of equilibrium				
first period	pooling	separating	pooling or separating	pooling or separating
second period	pooling	separating	separating	separating
Consumer lock-in	yes	no	yes when pooling in the first period*	no
Temporal profit pattern				
low risks	lowballing	highballing	lowballing	highballing**
high risks	for portfolio	flat	for portfolio*	undeterminate or flat**

* The pooling equilibria attributed to Nilssen is that in the paper's theorem. This result is a possible equilibrium and the lowballing prediction applies to this equilibrium. He points out that other equilibria can exist.

** When pooling in the first period. The profit pattern for high risks who pool is indeterminate. The pattern for those who do not pool is flat. Overall, the temporal profit pattern for high risks depends on the proportion who pool, i.e. on "x".

TABLE 2

LOSS/PREMIUMS = a + b[PREMIUM GROWTH] + c[MARKETING EXPENSES] + d[LIABILITIES/PREMIUMS] + e[DIRECT WRITER DUMMY] + f[REINSURER]

INSURANCE LINE	COEFFICIENT a	b	c	d	e	f	Adjusted R^2
All lines	0.5805	0.0747	0.0584	-0.3198	0.0839	0.0811	0.318
	28.98	4.829	7.597	-5.723	4.079	1.689	
Automobile Liability	0.7249	0.0461	0.3378		0.1016	0.0672	0.144
	21.89	6.601	1.989		3.091	1.324	
Automobile Physical damage	0.5788	0.0346	-0.2673		0.0568	-0.0294	0.132
	26.26	7.000	-0.770		2.604	-0.800	
Medical Malpractice	1.0132	-0.0342	-1.5661		-0.091	0.5926	0.093
	7.926	-0.701	-1.863		-0.673	2.752	
Fire	0.4781	0.0288	-0.2673		0.0683	0.1416	0.137
	18.84	5.362	-2.240		2.754	3.911	
Workers Compensation	0.8480	0.0241	-0.2467		0.0055	0.1321	0.021
	15.56	2.193	-0.806		0.118	1.945	
Homeowners	0.6321	0.0382	-0.2563		0.0505	0.0066	0.125
	23.69	5.727	-1.954		2.147	0.186	
Commercial Liability	0.5436	-0.0072	-0.3968		0.0755	0.4496	0.1224
	8.968	-0.627	-0.133		1.390	6.599	

TABLE 3

CALIFORNIA AUTOMOBILE LIABILITY RESULTS BY AVERAGE LOSS PER POLICY

	Intercept	Premium Growth	Dummy	Best's Rating	
1	-0.0969	-0.1036*	0.0437	0.0498	Lowest Average Loss
	(-0.597)	(-2.646)	(0.512)	(0.791)	↑
			ADJ R^2 0.18		⋮
2.	0.2027	0.0770	-0.0982	-0.0722	⋮
	(0.942)	(0.766)	(-1.327)	(-1.101)	⋮
			ADJ R^2 0.01		⋮
3.	-0.1809	0.2429*	0.0924	0.0771	↓
	(-0.367)	(2.219)	(0.703)	(1.565)	Highest Average Loss
			ADJ R^2 0.23		

Notes. - * significant at 0.05
- Dummy = 1 for stock; = 0 for mutual
- Best's Rating variable inserted since high loss to premium ratios might simply reflect poor financial and underwriting management. Poor management might also result in negative growth as such firms withdraw from the market.

FIGURE 1

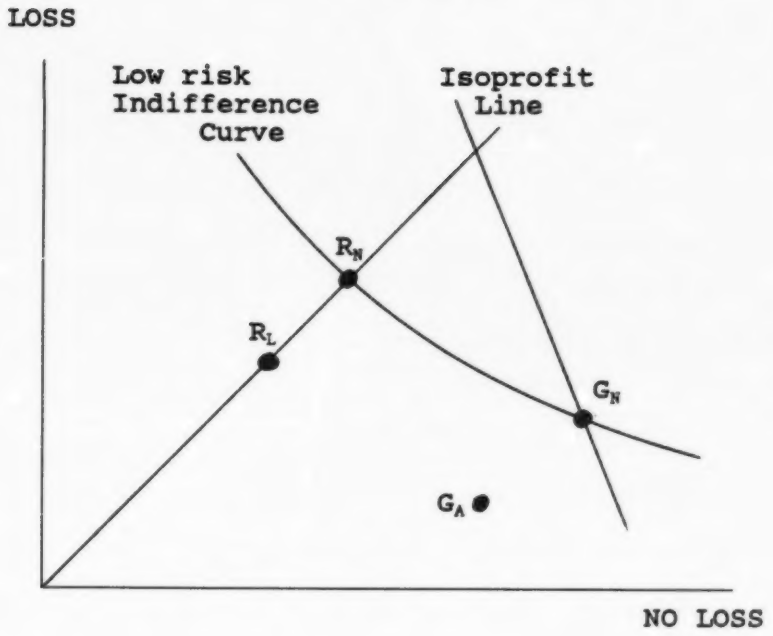


FIGURE 2

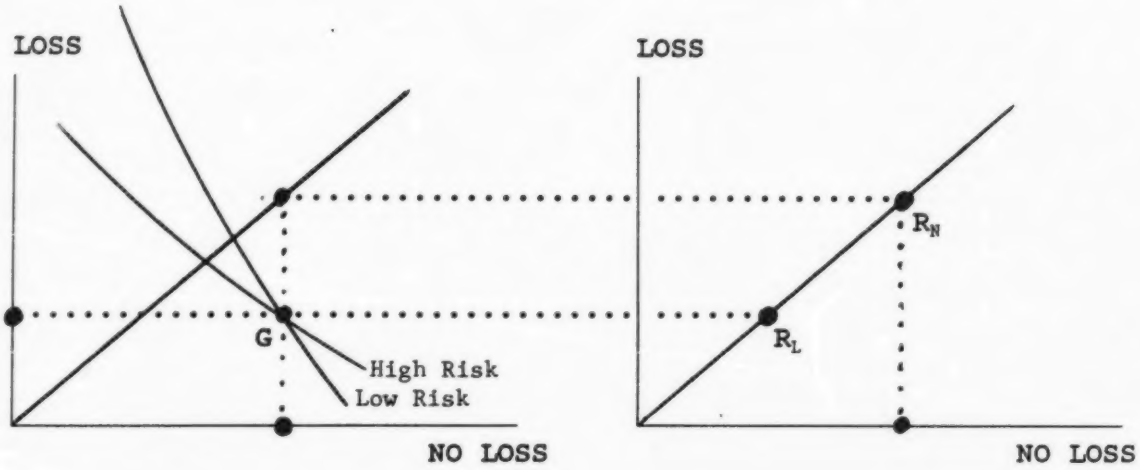


FIGURE 3

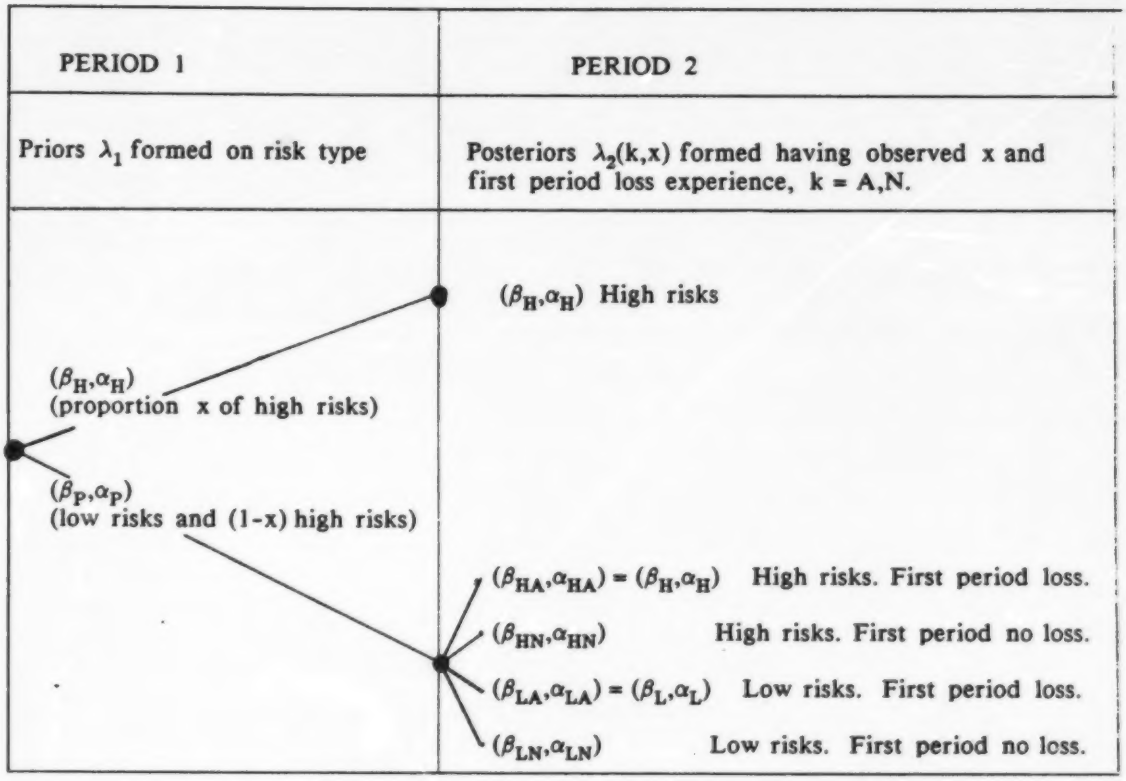
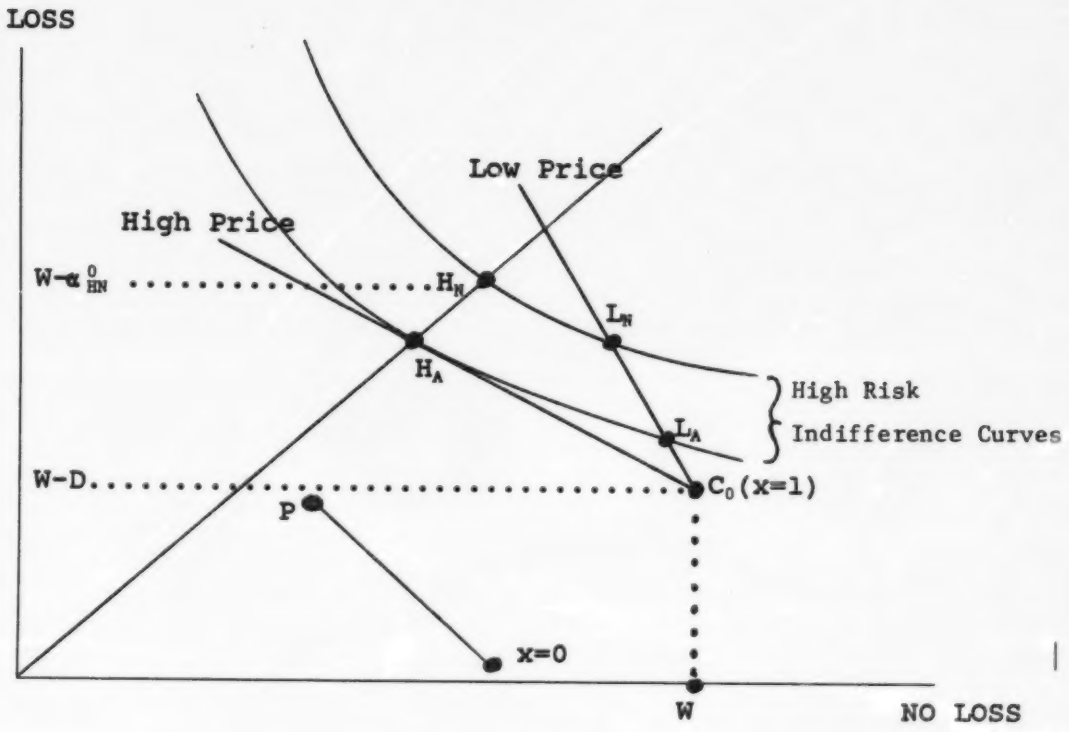

PERIOD 1
PERIOD 2
Priors λ_1 formed on risk type
Posteriors $\lambda_2(k,x)$ formed having observed x and first period loss experience, k = A,N.
(β_H,α_H) High risks
(β_H,α_H)
(proportion x of high risks)
(β_P,α_P)
(low risks and (1-x) high risks)
$(\beta_{HA},\alpha_{HA}) = (\beta_H,\alpha_H)$ High risks. First period loss.
(β_{HN},α_{HN}) High risks. First period no loss.
$(\beta_{LA},\alpha_{LA}) = (\beta_L,\alpha_L)$ Low risks. First period loss.
(β_{LN},α_{LN}) Low risks. First period no loss.

FIGURE 4



where:

$H_A = (\beta_H, \alpha_H)$
$H_N = (\beta_{HN}, \alpha_{HN})$
$L_A = (\beta_{LA}, \alpha_{LA})$
$L_N = (\beta_{LN}, \alpha_{LN})$